Frequency-based redshift for cosmological observation and Hubble diagram from the 4-D spherical model in comparison with observed supernovae

DICE2016, Sep. 12 - 16, Castiglioncello (Italy)
Shigeto Nagao, Ph.D.
(Osaka, Japan)

Summary
- According to the formerly reported 4-D spherical model of the universe, factors on Hubble diagrams are discussed.
- The observed redshift is not the prolongation of wavelength from that of the source at the emission but from the wavelength of spectrum of the present atom, equal to the redshift based on the shift of frequency from the time of emission.
- The K-correction corresponds to conversion of the light propagated distance (luminosity distance) to the proper distance at present (present distance).
- Comparison of the graph of the present distance times $1+z$ versus the frequency-based redshift with the reported Hubble diagrams from the Supernova Cosmology Project, which were time-dilated by $1+z$ and K-corrected, showed an excellent fit for the Present Time (radius of 4-D sphere) being 0.7 of its maximum.
4-D Spherical Model of the Universe

- The space energy spreads with expansion in a 3-D surface of a 4-D sphere. A vibration of the intrinsic space energy in the 3-D space vests additional energy, which is light or a quantum particle. Radius \( x \) of the 4-D sphere is our “Observed Time” (Time or \( T \)), which we feel passing commonly for all of us.

\[
x: \text{Radius vector of 4D sphere} \quad x = (x, \theta) = (x, \theta_1, \theta_2, \theta_3) \quad (4D \text{ spherical coordinate})
\]

\[
r: 3D \text{ space vector corresponding to } \theta \quad r = (r, \varphi_1, \varphi_2) = (x \theta, \varphi_1, \varphi_2)
\]

\[
x \text{ by 4D cylindrical coordinate: } \quad x = (x, r, \varphi_1, \varphi_2) = (x, x \theta, \varphi_1, \varphi_2)
\]

< Definition of terms related to Time >

- \( x \) (Radius of universe): Radius of the 4D sphere of universe, equal to the Observed Time (\( T \))
- \( \text{CU (Cosmic Unit):} \) Unit of \( x \) and \( T \), being one at its maximum when the space expansion stops. Time-related variables are expressed in the CU in this article.

- \( T_E \) (Time of Emission): Time when the light was emitted.
- \( T_P \) (Present Time): Present Time of universe, when the light reaches us.
- \( T_{ER} \) (Relative Time of Emission): Relative ratio of \( T_E \) to \( T_P \). \( T_{ER} \equiv T_E / T_P \)
- \( T_B \) (Back in Time): Back in Time from present when the light was emitted. \( T_B \equiv T_P - T_E \)
- \( T_{BR} \) (Relative Back in Time): Relative ratio of \( T_B \) to \( T_P \). \( T_{BR} \equiv T_B / T_P \)
- \( T_C \) (Time Clear): Time when the space became transparent to light.
- \( T_{CR} \) (Relative Time Clear): Relative ratio of \( T_C \) to \( T_P \). \( T_{CR} \equiv T_C / T_P \)

DICE2016, Castiglioncello (Italy)
Redshift for cosmological observation

We compare the observed wavelength $\lambda(T_P)$ with that of the present atom $\lambda_0(T_P)$.

\[ \nu_0(T_E), \lambda_0(T_E) \rightarrow \nu(T_P), \lambda(T_P) \]

\[ \nu_0(T_E) = \nu_0(T_P) \quad \nu_0(T_P), \lambda_0(T_P) \text{ (present atom)} \]

**Observed redshift** $z$:

\[ z + 1 = \frac{\lambda(T_P)}{\lambda_0(T_P)} = \frac{\nu_0(T_P)}{\nu(T_P)} = \frac{\nu_0(T_E)}{\nu(T_P)} \text{ equal to frequency-based redshift } z_\nu. \]

**Wavelength-based redshift:**

- **From space expansion:** Light speed does not vary. Wavelength is stretched by $n$.
- **From light speed variation:** Wavelength prolongs in proportion to the speed.

\[ z_\lambda = 1 + \frac{\lambda(T_P)}{\lambda_0(T_E)} = n \times \frac{C(T_P)}{C(T_E)} = \frac{T_P}{T_E} \times \frac{C(T_P)}{C(T_E)} = \frac{1}{T_{ER}} \times \frac{C(T_P)}{C(T_E)} \]

**Frequency-based redshift:**

\[ z_\nu = 1 + \frac{\nu_0(T_E)}{\nu(T_P)} = \frac{\lambda(T_P)}{\lambda_0(T_P)} = \frac{\lambda(T_P)}{\lambda_0(T_E)} \frac{\lambda_0(T_P)}{\lambda_0(T_E)} = n \times \frac{C(T_P)}{C(T_E)} \times \frac{C(T_E)}{C(T_P)} = n = \frac{1}{T_{ER}} \]

\[ z_\nu = z = \frac{1}{T_{ER}} - 1 \]

DICE2016, Castiglioncello (Italy)

3
Magnitude of light propagated distance \(LD\) to \(z=0.05\)

Light of luminosity \(L\) emitted at \(T_E\) reaches us now at \(T_P\).

Flux we observe now: \(F(T_E) = \frac{L}{4\pi \cdot LD(T_E)^2}\) Its magnitude: \(m(T_E) = -2.5 \cdot \log F(T_E)\)

Relative magnitude to same luminosity at \(z=0.05\): \(DM_{0.05}(T_{ER}) = m(T_{ER}) - m(z = 0.05) = m(T_{ER}) - m(1/1.05)\)

Light speed: \(C(x) = K \cdot f_D \cdot f_{EM} = K \cdot \frac{1}{x\sqrt{1-x}} \cdot \left(1 - \frac{T_C^3}{x^3}\right)\)

Light propagated distance: \(LD(T_E) = \int_{T_E}^{T_P} C(x) dx = K \left(\log \left(\frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}}\right) - \log \left(1 + \sqrt{1-T_E}\right)\right)\)

\(DM_{0.05}(T_{ER}) = 5 \cdot \log \left(\frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \cdot \frac{1 + \sqrt{1-T_P T_{ER}}}{1 - \sqrt{1-T_P T_{ER}}}\right) - 5 \cdot \log \left(\frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \cdot \frac{1 + \sqrt{1-T_P / 1.05}}{1 - \sqrt{1-T_P / 1.05}}\right)\), a distance modulus

DICE2016, Castiglioncello (Italy)
Hubble diagram

3-D space expansion speed: \[
\frac{dr}{dT} = \frac{dr}{dx} = \frac{d(x\theta)}{dx} = \theta \quad r = \theta \cdot x
\]

For a given angle \(\theta\), variable \(x\): constant \(\theta\)

For a given radius \(x\), variable \(\theta\): proportional to \(\theta\) and to \(r\) \(\rightarrow\) Hubble’s law

Hubble diagram: To discuss the recessive velocity of the proper distance

Tentatively provide that light speed has been constant.

Light propagated distance: \(LD_C = c \cdot (T_p - T_E) = c \cdot T_p \cdot (1 - T_{ER}) = c \cdot T_p \frac{z}{z + 1}\)

Multiply by \((1 + z)\) (time dilation): \((1 + z) \cdot LD_C = c \cdot T_p \cdot z = k \cdot z \rightarrow\) proportional to \(z\)

\(\Delta t' = (1 + z)\Delta t\) Light-curve width of supernovae: dilated by \(1 + z\)

Ratio of PD to LD (LD-PD conversion):

- \(A_P - B_P\): proper distance at \(T_P\) (“present distance, PD”)
- \(C - B_P\): light propagated distance (LD)
  (= 3D-space component of \(A_EB_P\))

\[
\begin{align*}
n &= T_p / T_E = 1 / T_{ER} \\
\frac{CB_p}{A_EB_E} &= 1 + \frac{1}{2} (n - 1) = \frac{1}{2} (n + 1), \quad \frac{PD}{LD} &= \frac{2n}{n + 1} = \frac{2(z + 1)}{z + 2} = \frac{2}{1 + T_{ER}}
\end{align*}
\]
**Adjusted magnitude of present distance PD to z=0.05**

For comparison with reported Hubble diagrams, take the following adjusted PD:

\[
(1 + z) \cdot PD = (1 + z) \cdot \frac{2(z + 1)}{z + 2} \cdot LD \quad \text{or} \quad \frac{1}{T_{ER}} \cdot PD = \frac{1}{1 + T_{ER}} \cdot \frac{2}{1} \cdot LD
\]

*Time dilation  LD-PD conversion*

Reported Hubble diagrams from the SCP: *Time-dilated* and *K-corrected* \(D_L\) (luminosity distance)

**K-correction** \(K_{xy}\):

\[
m_y = M_x + DM^0 + K_{xy}, \quad m_y = M_y + DM(z) \quad \rightarrow \quad K_{xy} = M_y - M_x + DM(z) - DM^0
\]

1) cross-filter adjustment on absolute magnitude \(M_y - M_x\), plus

2) difference in distance modulus \(DM(z) - DM^0\) (= mag LD – mag PD) between observed / rest frames

Frame-conversion part of **K-correction** corresponds to **LD-PD conversion**.

**Adjusted magnitude of PD to z = 0.05**, \(DM_{0.05}^{adj}\): Add *Time dilation* and **LD-PD conversion** to \(DM_{0.05}\)

\[
DM_{0.05}^{adj}(T_{ER}) = 5 \cdot (\lg LD(T_{ER}) + \lg(1/T_{ER}) + \lg(2/(1 + T_{ER}))) - \lg LD(1/1.05) - 1 \cdot \lg 1.05 - \lg(2 \cdot 1.05/2.05)
\]

\[
= 5 \cdot (\lg LD(T_{ER}) - \lg(T_{ER}) - \lg(1 + T_{ER}) - 1 \cdot \lg LD(1/1.05) - 2 \cdot \lg 1.05 + 1 \cdot 2.05)
\]

\[
DM_{0.05}^{adj}(T_{ER}) = 5 \cdot \log \left(\frac{1 - \sqrt{1 - T_P}}{1 + \sqrt{1 - T_P}T_{ER}} \cdot \frac{1 + \sqrt{1 - T_P}}{1 - \sqrt{1 - T_P}T_{ER}}\right) - 5 \cdot \left[\log \left(\frac{1 - \sqrt{1 - T_P}}{1 + \sqrt{1 - T_P} / 1.05} \cdot \frac{1 + \sqrt{1 - T_P / 1.05}}{1 - \sqrt{1 - T_P / 1.05}}\right)\right]
\]

\[
- 5 \cdot \lg T_{ER} - 5 \cdot \lg(1 + T_{ER}) - 10 \cdot \lg 1.05 + 5 \cdot \lg 2.05
\]

DICE2016, Castiglioncello (Italy)
The adjusted magnitude of PD to $z = 0.05$, $DM_{0.05}^{adj}$, versus the redshift $z = 1/T_{ER} - 1$

---

- $T_p = 0.6$
- $T_p = 0.7$
- $T_p = 0.8$

- - - - : Constant light speed (for any $T_p$)

---

- - - - : reference $DM_{0.05}^{adj}$ based on $LD_C = c \cdot T_P \left(1 - T_{ER}\right)$ subject to constant light speed
Comparison with Hubble diagrams from the SCP

Superimposition on the Hubble diagram by Perlmutter et al, $z$ in a logarithmic scale


Calan/Tololo (Hamuy et al, A.J. 1996)

$\Omega_M, \Omega_{\Lambda} = (0, 0)$
$\Omega_M, \Omega_{\Lambda} = (1, 0)$
$\Omega_M, \Omega_{\Lambda} = (1.5, -0.5)$

$T_p = 0.6$
$T_p = 0.7$
$T_p = 0.8$

Adjusted magnitude of PD to $z=0.05$, $DM_{0,05}^{\text{effective}}$
Superimposition on the latest Hubble diagram from the SCP, $z$ in a uniform scale


Adjusted magnitude of PD to $z=0.05$, $D_M^{adj, 0.05}$

Distance Modulus

Redshift z

Redshift 1.71 Supernova

This Work

$T_P = 0.6$

$T_P = 0.7$

$T_P = 0.8$

Rodney et al. (2012)
Suzuki et al. (2012) (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)

Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)

Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisiunas et al. (2005)
Hannay et al. (1996)
Conclusion

- Observed redshift is the frequency-based redshift from the time of emission.
- Light propagated distance LD is equal to the luminosity distance.
- Ratio of converting LD to the present distance PD (proper distance at present) is given.
- The frame-conversion part of the K-correction corresponds to the LD-PD conversion.
- Magnitude of \((1 + z) \cdot PD\) to \(z = 0.05\) was compared with reported Hubble diagrams from the SCP, which were time-dilated by \(1 + z\) and K-corrected.
- Superimposition on the reported Hubble diagrams from the SCP showed an excellent fit. The graph for the Present Time \(T_p = 0.7\) very closely overlaps the line of flat \(\Omega_m = 0.27\) \(\Lambda\)CDM universe that Rubin et al concluded as the best fit.